Krivulje - Curves

1. Introduction

Osnovni cilj ovog poglavlja

1.1 Representation of Curves

D. F. Rogers and J. A. Adams, *Methematical Elements for Computer Graphics*, McGraw-Hill Book Company, New York, 1976, Chapter 5 Space Curves, p. 136-138:

Space curves may be represented either **nonparametrically or parametrically**. Three-dimensional space curves expressed in nonparametric form are given explicitly by a set of equations of the form:

$$\begin{aligned} x &= x \\ y &= f(x) \\ z &= g(x) \end{aligned} \tag{1}$$

Alternately a space qurve my be expressed in a nonparametric, implicit form. In this case the space curve is represented mathematically by the intersection of the two surfaces given by:

$$\begin{aligned} f(x,y,z) &= 0 \\ g(x,y,z) &= 0 \end{aligned} \tag{2}$$

In general, a parametric space curve is expressed as:

$$\begin{aligned} x &= x(u) \\ y &= y(u) \\ z &= z(u) \end{aligned} \tag{3}$$

where the parameter u varies over a given range $u_1 \leq u \leq u_2$. Reconsidering Eq. (1) we see that x itself can be considered a parametar, $u = t$, and the same is then expressed in parametric form by:

$$\begin{aligned} x &= u \\ y &= y(u) \\ z &= z(u) \end{aligned} \tag{3}$$

When an analytical description for a curve is not known, an interpolation sheme my be used to fit a curve through a given set of data points. This involves specifying boundary conditions for the space curve in order to determine the coefficients for a given polynomial curve form and establishing a smoothness criterion.

When the input is a series of points which lie on the desired curve, spline segments are often used to form a smooth curve through the points.

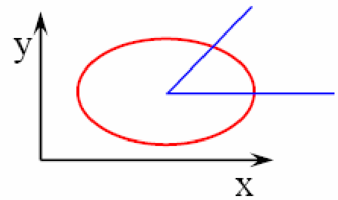
Representation of Curves:

a) eksplicitni oblik - nemogućnost prikaza višestrukih vrijednosti

$$y = f(x), \qquad z = g(x)$$

b) implicitni oblik - za prikaz dijela krivulje trebaju dodatni uvjeti

$$F(x,\ y,\ z) = 0$$

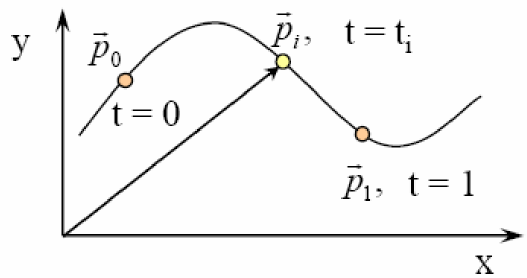


c) parametarski oblik

$$x = \mathrm{x}(t), \quad y = \mathrm{y}(t), \quad z = \mathrm{z}(t).$$

točka na krivulji - vektorska funkcija

$$\mathrm{V}(t) = [\mathrm{x}(t) \quad \mathrm{y}(t) \quad \mathrm{z}(t)].$$



vektor tangente

$$\mathrm{V}'(t) = [\mathrm{x}'(t) \quad \mathrm{y}'(t) \quad \mathrm{z}'(t)].$$

$$\mathrm{V}(t_i) = \vec{p}(t_i) = \vec{p}_{t_i}.$$

Two types of equations for curve representation

(1) Parametric equation

x, y, z coordinates are related by a parametric variable $(u \text{ or } \theta)$

(2) Nonparametric equation

x, y, z coordinates are related by a function

Example: Circle (2-D)

Parametric equation

$$x = R\cos\theta, \quad y = R\sin\theta \quad (0 \le \theta \le 2\pi)$$

Nonparametric equation

$x^2 + y^2 - R^2 = 0$ **(Implicit nonparametric form)**

$y = \pm\sqrt{R^2 - x^2}$ **(Explicit nonparametric form)**

Two types of curve equations

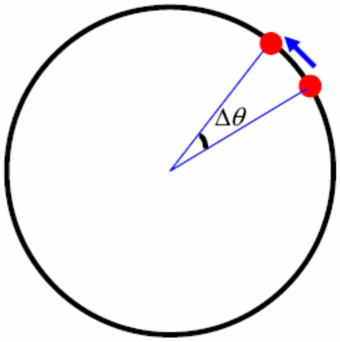
(1) Parametric equation Point on 2-D curve: $\mathbf{p} = [x(u) \quad y(u)]$

Point on 3-D surface: $\mathbf{p} = [x(u) \quad y(u) \quad z(u)]$

u : parametric variable and independent variable

(2) Nonparametric equation $y = f(x)$: 2-D , $z = f(x, y)$: 3-D

Which is better for CAD/CAE? : Parametric equation



$$x = R\cos\theta, \quad y = R\sin\theta \quad (0 \le \theta \le 2\pi)$$

$$x^2 + y^2 - R^2 = 0$$

$$y = \pm\sqrt{R^2 - x^2}$$

It also is good for calculating the points at a certain interval along a curve.

Notes about Curves:

– analitički izraz izvorne krivulje u pravilu je nepoznat

– poznato je
  - koordinate u nekim točkama
  - nagibi, zakrivljenost ili izvijanje u nekim točkama

  ⇒ modeliranje

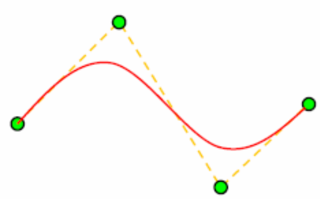
– opis segmenta krivulje

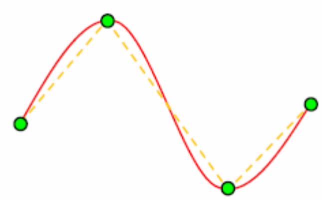
– segmentiranje

  - povezivanje segmenata uz ostvarivanje kontinuiteta između segmenata

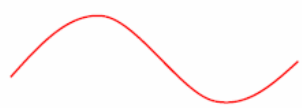
1.2 Podjela krivulja:

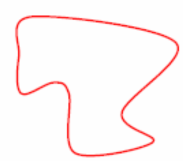
- aproksimacijske
- interpolacijske





- otvorene
- zatvorene





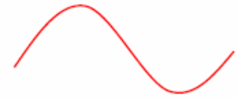
- razlomljene
- nerazlomljene

$$x(t)=\frac{a_1t^3+b_1t^2+c_1t+d_1}{at^3+bt^2+ct+d}$$

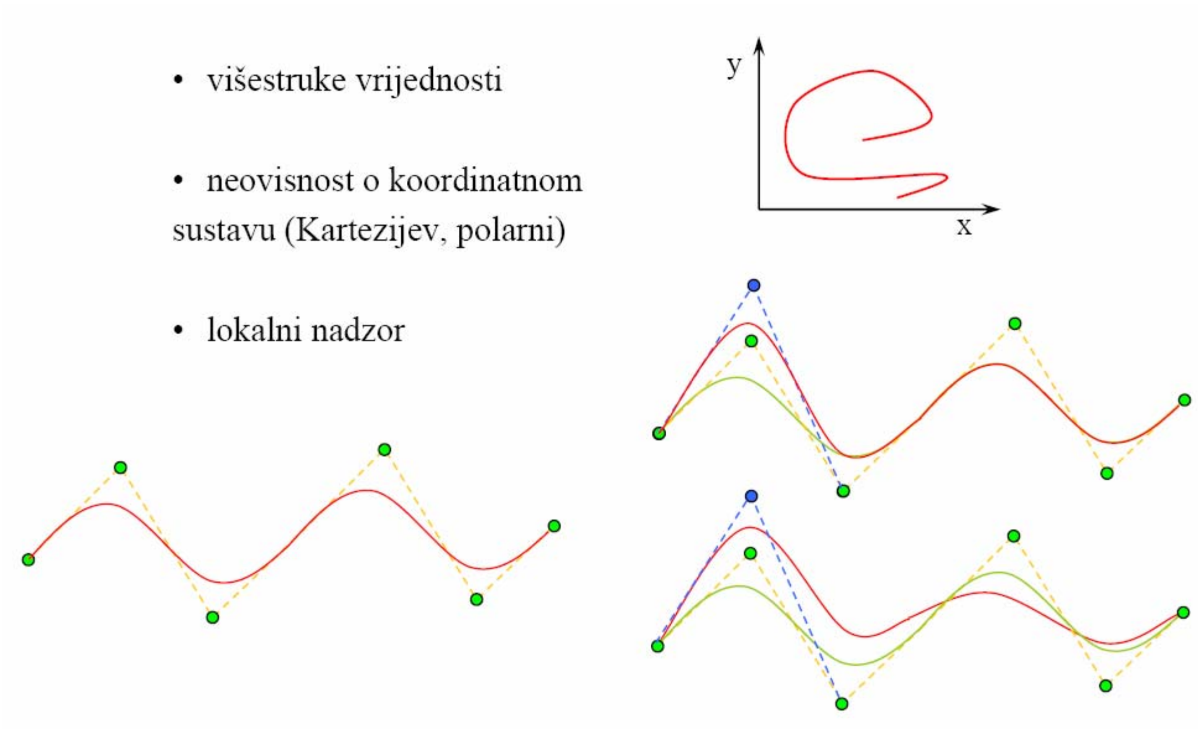
$$x(t)=a_1t^3+b_1t^2+c_1t+d_1$$

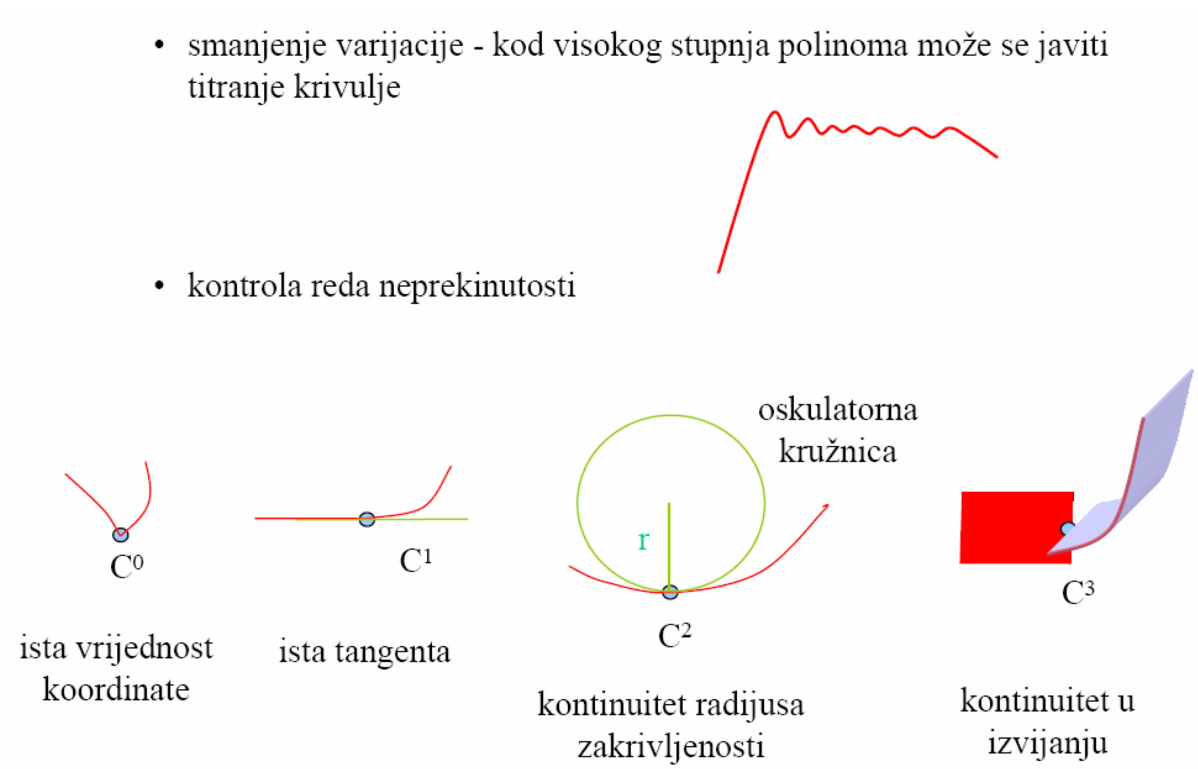
- periodične
- neperiodične

(periodičnost težinskih funkcija)



1.3 Poželjna svojstva krivulja:





C^0 - ista vrijednost koordinate $f(t) = g(t)$

C^1 - ista vrijednost derivacije $f'(t) = g'(t)$

C^2 - ista vrijednost druge derivacije $f''(t) = g''(t)$

Zakrivljenost krivulje obrnuto je proporcionalna radijusu oskulatorne kružnice. Ako je radijus velik zakrivljenost je mala (i obrnuto).

C^3 - ista vrijednost treće derivacije $f'''(t) = g'''(t)$

Osim C kontinuiteta postoje i G kontinuiteti koji zahtijevaju proporcionalnost.

G (geometrijski)

G^1 - proporcionalna vrijednost derivacije $f'(t) = k_1\, g'(t), k_1>0$

G^2 - proporcionalna vrijednost druge derivacije $f''(t) = k_2\, g''(t), k_2>0$

G^3 - proporcionalna vrijednost treće derivacije $f'''(t) = k_3\, g'''(t), k_3>0$

C^1 kontinuitet implicira G^1 kontinuitet osim kada je vektor tangente [0 0 0]

kod C^1 kontinuiteta može doći do promjene smjera, kod G^1 ne može.